Some Guidelines for Evaluating Capital Investment Alternatives with Unequal Lives

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Introduction

The necessity of making a choice between mutually exclusive investment alternatives that have unequal lives is a problem frequently encountered in practice. It arises in considering the replacement of existing equipment and in considering alternative ways to perform a new task. There is disagreement as to how this problem should be addressed, however.

The two procedures suggested most frequently in the capital budgeting literature are the "study period" and "common life multiple" methods. (See Bierman and Smidt [1], Grant, Ireson and Leavenworth [2], Osteryoung [4], and Taylor [6].) Both procedures require two steps. The first step for each is specifying the interval of time over which the investment alternatives will be evaluated. The study period approach restricts the analysis to an evaluation interval (study period) usually defined as the life of the shortest-lived alternative. The common life multiple technique employs an evaluation interval equal to the lowest common multiple of the lives of the alternatives under consideration.

The second step is determining each investment alternative's cash flows for the chosen evaluation interval. The study period method implicitly includes cash flows for only the first generation of equipment. The common life multiple procedure incorporates cash flows of future generations of equipment, with the usual assumption that future equipment will entail the same costs and benefits as equipment currently available (*i.e.*, like-for-like replacement).

Both techniques have deficiencies at both steps. First, the choice of the evaluation interval is arbitrary. The study period is an arbitrarily short interval which ignores the cash flows that occur subsequent to the conclusion of the study period. The common life multiple may be an arbitrarily long interval which includes

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future cash flows that should have no influence upon the project currently under consideration. Second, the common life multiple assumption of like-for-like replacement is unwarranted because of the possibility of technological advances.

The purpose of this paper is to provide guidelines for the analysis of mutually exclusive investment alternatives with unequal lives, which should eliminate the deficiencies already noted. I have restricted my attention to new proposals, for the replacement problem has been examined in detail elsewhere. (See Grant, Ireson, and Leavenworth [2] and Taylor [6].)

A Classification of Problems with Unequal Lives

The proper use of a discounted cash flow decision criterion for making a choice between mutually exclusive investment alternatives requires that the alternatives be comparable. Investments with unequal lives may violate this requirement in some but not all cases. Whether the requirement is violated depends on the relationship between the life of the task that is to be performed (the project) and the lives of the mutually exclusive methods of performing this task (the alternatives).

The enumeration of all possible relationships between the project life and the lives of the alternatives provides the classification of problems with unequal lives. This classification scheme serves two purposes: it reveals when unequal-lived alternatives are and are not comparable, and it indicates the length of the interval over which they should be evaluated. There are four cases to consider.

Case 1. Project Life Determined by Alternative Life

The life of the project is determined by and coincides with the life of the selected alternative for Case 1. This may be due to an arbitrary management decision to carry out the activity only as long as one generation of equipment will provide service. More likely, it is the result of the economic characteristics of the project.

The extraction of a fixed quantity of some natural resource (say, coal) is an example of this type of project. The alternatives to consider may be a process that requires ten years to recover the coal and a second process that can accomplish the task in only eight years. There is no opportunity to continue the project if the short-lived process is employed for all the coal will have been retrieved. The alternatives currently under consideration completely describe the future outcomes associated with this project. The choice between the alternative processes can be made strictly on the basis of the costs and the timing of the benefits associated with each one. In other words, a common evaluation interval is not necessary; each alternative may be evaluated over its own particular life.[1]

The project life is independent of the lives of the alternatives in Cases 2 through 4. While the project life may be established by a managerial decision regarding how long the firm will produce the product, a different procedure is required to determine the life of any particular alternative. Explicitly or implicitly, an alternative's life should be its *economic* life. This life is based on the replacement schedule that provides for the lowest cost performance of the task to which the equipment will be applied.

Case 2. Project Life Independent of and Shorter Than the Lives of the Alternatives

This situation arises whenever the firm ceases to perform a specific task even though the equipment used in the process is still serviceable. The construction industry provides a good example of this case. The construction of a particular building may require only two years, while all the cranes the firm can choose have lives that exceed two years. No future decision involving this project depends upon the choice among these alternatives, however, because the task will be completed prior to the expiration of their lives. The alternatives are directly comparable even though their lives are unequal. The analyst does have the difficult problem of estimating salvage values at the termination of the project, but in all other respects, the firm conducts its evaluation *as if* each alternative has a life equal to the life of the project.

Case 3. Project Life Independent of and Longer Than the Lives of the Alternatives

This situation may be the one most frequently encountered in practice. It occurs because the performance of some activity requires the periodic replacement of equipment in order to keep costs to the minimum. Consider a firm that expects to produce a particular model of aircraft for many years but must choose between two types of rivet guns with economic

[1]The Lorie and Savage [3] and Solomon [5] pump problem fits this case also although it involves a replacement problem rather than a new project. Solomon was correct not to equalize the lives of the pumps because the life of the project depended upon which pump was used.

lives of two and three years.

Future decisions are influenced by the current choice in this case, because replacements will be required regardless of which alternative is selected initially. The analyst must therefore evaluate the *sequence* of decisions associated with each alternative. This evaluation must encompass an interval beyond which future events are not influenced by the current choice. An evaluation interval set equal to the life of the project obviously meets this requirement. The lowest common multiple of the lives of the alternatives also represents an interval at the conclusion of which the firm has identical options regardless of which choice was made initially. The actual evaluation interval employed should be the lesser of the project life or the lowest common multiple.

Case 4. Project Life Independent of and Between the Lives of the Alternatives

Case 4 may be thought of as a combination of Cases 2 and 3. The firm intends to perform some task for a specific period of time, and has alternatives whose lives span that period. A firm that requires the use of a warehouse for fifteen years, for example, may choose between a temporary air suspension facility with a life of five years or a conventional building that is a permanent structure. No future replacement is required if the permanent warehouse is constructed, so it should be evaluated as if its life is equal to the life of the project. (As always, it is necessary to recognize its residual value at the termination of the project.) On the other hand, a sequence of decisions is associated with the temporary warehouse because it must be replaced if service is to continue over the life of the project. The proper evaluation interval for both alternatives is therefore the project life even though this requires only a single purchase for one alternative and a series of replacements for the other. Note that the study period method may be applied to a Case 4 problem if the life of the shortest-lived alternative coincides with the life of the project.

In summary, not all investment problems with unequal lives require the specification of an evaluation interval. No adjustments are required to evaluate the alternatives in Cases 1 and 2 because the life of the project is such that no future decision depends upon the current choice. Evaluation intervals must be specified for Case 3 and Case 4 problems, however, because the life of the project necessitates the consideration of future replacement decisions. The lowest common multiple and study period procedures may provide the proper evaluation interval for some Case 3 and Case 4 problems, respectively, but neither procedure is always correct.

Compiling the Cost Schedules

The next step in the analysis of Case 3 and Case 4 problems is determining the cash flows associated with acquiring and operating future generations of equipment. Only the cost cash flows are considered here, because the revenue cash flows are assumed to depend on the task to be performed (the project) rather than the alternative that is selected. Revenue cash flow schedules may be added if they differ for each alternative. As with all capital budgeting problems, the estimation of costs is more difficult in the presence of inflation. I have ignored the problem of inflation in order to emphasize the solution to the unequal lives problem.

Other authors have proposed simplified procedures, such as the assumption of like-for-like replacement, because of the difficulty of determining the costs of acquiring and operating future generations of equipment. The very fact that an alternative's economic life has been specified, though, implies that the analyst has at hand the information necessary to determine these costs more accurately. This procedure is discussed and illustrated following a review of the determination of economic life.

Determining Economic Life

The *economic* life of a piece of equipment is the holding period during which the equipment can perform a particular task at minimum uniform annual total cost. This cost, which depends on the characteristics of the asset and the context in which it is employed, has two components. One component, the uniform equivalent capital cost, is the annualized cost of purchasing the asset and its future replacements. The other component is the uniform annual equivalent cost of operating the asset and its future replacements. The length of time the firm holds the asset affects these costs in different ways.

The uniform equivalent capital cost is determined by dividing the asset's purchase price by the appropriate interest factor for converting an annuity to its present value equivalent. Because this interest factor becomes larger the longer the time interval, the capital cost component decreases the longer the firm uses an asset. The capital cost component therefore tends to cause firms to keep equipment longer.

Two elements, deterioration and obsolescence, combine to cause operating costs to increase the longer the firm employs an asset. The cost of operating any piece of equipment increases over time because of a decline in serviceability as it ages. This increase in actual costs is said to be due to deterioration. There is also an opportunity cost associated with continuing to use a piece of equipment rather than replacing it. This cost arises because new generations of equipment have technological improvements that permit reduced operating costs relative to the costs incurred by continuing to use obsolete equipment. Both the increase in actual cost because of deterioration and the increase in opportunity cost due to obsolescence tend to cause firms to replace equipment frequently. These relationships and the resulting uniform annual total cost curve are illustrated in Exhibit 1.

The form of these cost functions must be specified to determine an asset's economic life. The well accepted method of defining these functions assumes that the purchase price and salvage value of current and future generations of equipment are constant while deterioration and obsolescence increase arithmetically. (See Grant, Ireson, and Leavenworth [2] and Taylor [6].) The assumption of arithmetic deterioration means that the cost of operating any generation of equipment increases by a constant amount (G_d) each year. Thus, if D represents the cost of operating the equipment the first year of its life, the costs the second, third, and N^{th} years are $D + G_d$, $D + 2G_d$, and $D+(N-1)G_d$, respectively. The assumption of arithmetic obsolescence means the cost of operating improved equipment decreases by a constant amount (G_o) each year. The opportunity cost of not replacing the existing equipment is therefore G_o, $2G_o$, and NG_o for years one, two, and N, respectively.

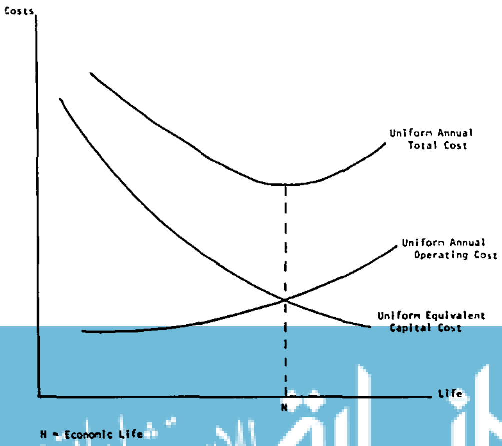
Note that the obsolescence gradient provides a description of how the operating cost behavior of current and future replacement equipment will differ. Future generations of equipment will be equivalent to current equipment (like-for-like replacement) only if G_o is equal to zero. Otherwise, the cost of operating replacement equipment will reflect the effect of technological advance.

The uniform annual total cost function is obtained by converting these operating cost elements to their uniform annual equivalents and adding the uniform equivalent capital cost. Taylor derived the expression for economic life by setting the derivative of this function equal to zero and solving for N [6, p. 176]. His expression must be modified when the salvage value is greater than zero with the following result:

$$N \cong (P - S)i/(G_d+G_o) + [1 - (1+i)^{-N}]/i \quad (1)$$

where N = economic life;
P = purchase price;
S = salvage value;
i = required rate of return;
G_d = annual deterioration amount; and
G_o = annual obsolescence amount.

Exhibit 1. Uniform Annual Cost Functions



Determining the Implied Costs

The important insight to be gained from the preceding discussion is that *the specification of an alternative's economic life implicitly contains information about the cost of acquiring and operating its future replacements.* This information may be used to obtain the costs needed to make the decision between mutually exclusive alternatives. There are two situations to consider.

The first situation pertains to firms that actually use Equation (1) to establish an alternative's economic life. The same deterioration and obsolescence costs that were used to solve for N must be used to determine the relevant costs over the evaluation interval. The use of the like-for-like replacement assumption at this stage of the analysis is appropriate only if one rules out the possibility of technological advance ($G_o = 0$). The use of this assumption when $G_o > 0$ will cause the analysis to be internally inconsistent because the costs used to evaluate the mutually exclusive choice problem will be greater than the costs used to solve for the economic life.

The more interesting situation is presented by firms that do not use Equation (1) to establish an alternative's economic life but rely instead on their own past experience with similar equipment, the manufacturer's recommendations about replacement intervals, or some other heuristic to determine N. These firms may use Equation (1) (with N given) to obtain the *implied* operating costs represented by the deterioration and obsolescence factors.

This process is simplified by recognizing that the second term on the right hand side of Equation (1) is the interest factor that converts an annuity of N periods to its present value equivalent. Call this the annuity discount factor at 100i% for N periods ($ADF_{i,N}$). Equation (1) therefore may be written as

$$N = (P - S)i/(G_d+G_o) + ADF_{i,N} \qquad (2)$$

Rearranging terms,

$$G_d+G_o = (P - S)i/(N - ADF_{i,N}). \qquad (3)$$

Equation (3) may be used to obtain the sum of the implied deterioration and obsolescence costs. The analyst must know only the purchase price and salvage value of the asset, its estimated economic life, and the firm's required rate of return. Values of $ADF_{i,N}$ are widely available in tables. After judgment or past experience has been used to split this sum into its component parts, the resulting values may be employed to construct the cost schedule for the alternative and its future replacements. The following numerical example illustrates the process.

An Illustration

Consider a firm that must purchase a fork-lift truck for materials handling. The firm's required rate of return is 10%. The firm will require the use of a fork-lift for as long as it is in business, so the project life is indefinitely long. Past experience indicates that the two trucks under consideration (A and B) have economic lives of four and six years, respectively. This is therefore a Case 3 problem with an evaluation interval of twelve years, the lowest common multiple of four and six. The illustration will consider only truck A from this point on.

Fork-lift A costs $11,300, has a salvage value of $3000 at the end of its economic life, and costs $2,000 to operate the first year. Periodic replacement is caused primarily by deterioration rather than by obsolescence, so the analyst estimates

$$G_d/(G_d+G_o) = .9. \qquad (4)$$

Upon substituting the appropriate values in Equation (3) and using the relationship in Equation (4), the analyst obtains G_d = $900 and G_o = $100. These values and the other cost data for fork-lift A may now be employed to develop the cost cash flow schedules displayed in Exhibit 2.

The first schedule in Exhibit 2 provides the cash flows caused by replacement of the fork-lift every four years. The second schedule is the basic cost of operating the equipment. The changes in these operating costs due to deterioration and the use of improved equipment are entered separately on the third and fourth schedules.

The cost of operating any generation of equipment increases by $900 each year because of the effect of deterioration. The cost of operating improved equipment decreases at a constant rate of $100 per year, given the obsolescence factor. Replacement equipment is purchased at the end of its predecessor's economic life. The cost of operating the replacement is therefore 4($100) = $400 less per year than the cost of operating its immediate predecessor.

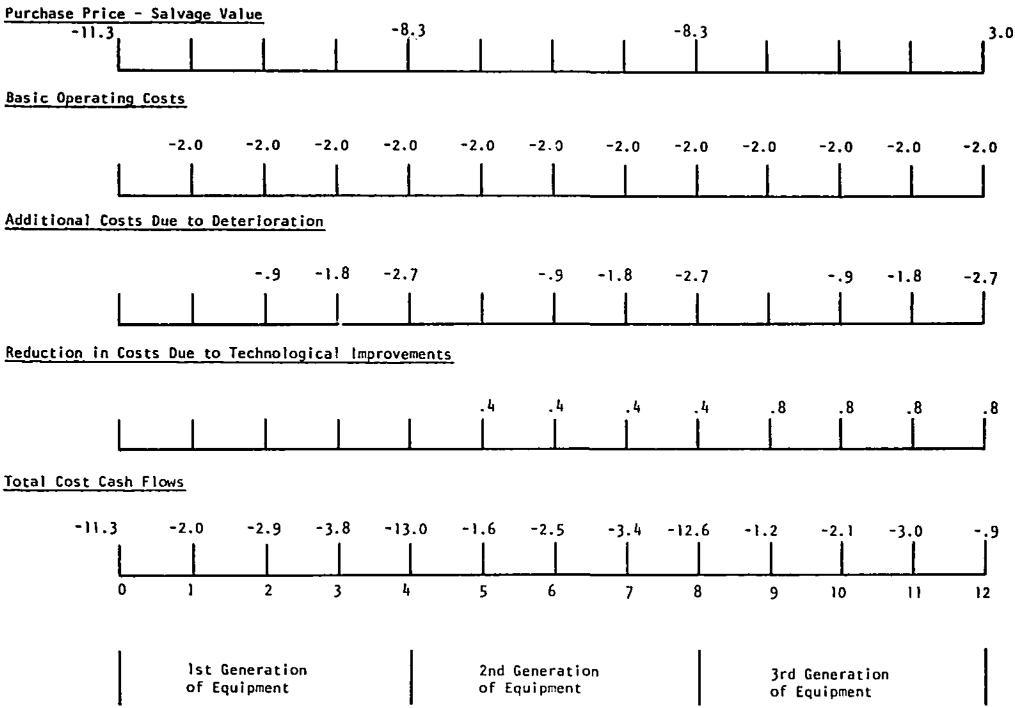
The total cost cash flow schedule is the period-by-period sum of the first four. These cash flows are shown on the final line of Exhibit 2. The analyst must repeat the process to derive a similar cash flow schedule for truck B.

The analyst may use this procedure to derive a cost cash flow schedule for each alternative associated with a Case 3 or Case 4 problem. Revenue schedules must be added if they are different for the alternatives; otherwise, the choice among alternatives may be based on their respective costs. The analyst may use any discounted cash flow decision criterion to make this choice once the appropriate cash flow schedules have been derived.

Summary

This paper has provided guidelines for the evaluation of mutually exclusive investment alternatives with unequal lives. The relationship between the life of the project and the lives of the alternatives was employed to classify investment problems with unequal lives. This taxonomy reveals when alternatives are or are not comparable and indicates the length of the interval over which they should be evaluated. The paper also showed that knowledge of an alternative's economic life provides information that may be used to determine the costs of acquiring and operating future generations of equipment. The procedures developed here eliminate the deficiencies found in the study period and common life multiple techniques.

Exhibit 2. Illustrative Cost Schedule ($000)



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